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ON PREDICTING BOILING BURNOUT FOR HEATERS COOLED BY LIQUID JETS*

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NOMENCLATURE

A,	area;
a,	exponent expressing We_f dependence of δ;
D,	disc diameter;
d,	jet diameter;
$\dot{E}$, $\dot{K}.\dot{E}.$,	rate of energy; rate of kinetic energy;
h_{fg},	latent heat of vaporization;
q_{max},	peak (or "burnout") boiling heat flux;
r,	ρ_f/ρ_g;
u_f,	liquid velocity;
$v_g, v_{g_{max}}$,	vapor velocity; maximum vapor velocity; $q_{max}/\rho_g h_{fg}$;
We_f,	"liquid" Weber number, $\rho_f u_f^2 D/\sigma$.

Greek symbols

α,	fraction of liquid flow directed into spray;
β,	D/δ;
δ,	surface area average droplet diameter;
ρ_f, ρ_g,	saturated liquid and vapor densities;
σ,	surface tension;
ϕ,	$(v_{g_{max}}/u_f) = q_{max}/\rho_g h_{fg} u_f$.

INTRODUCTION

Monde and Katto [1] recently provided an extremely successful correlation of peak heat flux (q_{max}) data for saturated liquid jets impinging on discs. Their configuration is shown in Fig. 1. They correlated 93% of 150 original and previous [2] q_{max} data for water and Freon 113, within $\pm 25\%$ of the best line through them. The correlation can be written as:

$$\phi = 0.0745 r^{0.725}/We_f^{1/3} \tag{1}$$

where ϕ is the vapor escape velocity at the peak heat flux, $v_{g_{max}}$, divided by the jet velocity, u_f. Thus $\phi \equiv q_{max}/\rho_g h_{fg} u_f$. The ratio of the saturated liquid and vapor densities is designated as r and $We_f \equiv \rho_f u_f^2 D/\sigma$, where D is the diameter of the heater.

More recently Katto and Ishii [3] achieved comparable success in correlating data for plane jets impinging at a 15° angle to a square heater. This time they based We_f on the length of the plate and found:

$$\phi = 0.0164 r^{0.867}/We_f^{1/3}. \tag{2}$$

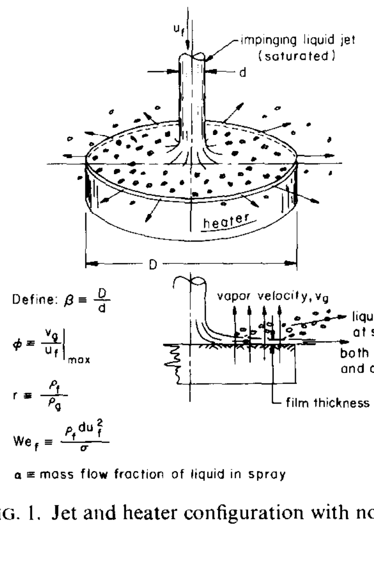

Fig. 1. Jet and heater configuration with nomenclature.

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In 1976, we addressed a different external flow-boiling burnout problem, namely that of predicting the peak heat flux during the cross-flow of saturated liquid over a horizontal cylindrical heater [4]. To predict q_{max} in that case we offered the following mechanical energy stability criterion:

The vapor removal wake must become unstable and collapse when the net mechanical energy added to it by the heater becomes positive.

In that flow configuration, vapor kinetic energy is added and newly created surface energy in the escaping vapor sheet consumes mechanical energy. When too much vapor kinetic energy is added to the sheet it becomes unstable and must widen abruptly to lower v_g. In so doing the sheet detaches from the leading edge of the cylinder, insulates it and brings about burnout.

Our present aim is to try to apply this criterion to the boiling configurations studied by Katto *et al.*, and in so doing to discover more about the physical mechanisms involved in jet-cooled heater burnout.

FORMULATION

We shall formulate a burnout prediction for the disc configuration shown in Fig. 1 and when it is complete we shall note any differences that might arise in the plane jet configuration. At burnout, the rate of mechanical energy supplied to the "wake" configuration equals that absorbed by it:

$$\dot{KE}_{\text{jet}} + \dot{KE}_{\text{vapor generated}} = \dot{E}_{\text{surface}} + \dot{KE}_{\text{liquid leaving}}. \quad (3)$$

The vapor generated next to the surface blows through the sheet, tearing holes in it. Its primary effect is to divide the liquid into two portions. One portion (with mass flow fraction, α) is a spray of droplets of average diameter,† δ, which absorbs a great deal of surface energy and departs at a slightly upward angle from the plane of the heater. The other portion (with mass flow fraction, $1-\alpha$), consists of the shards of the original sheet that remain attached to the heater and depart horizontally. We also presume that the velocity of both portions remains close to u_f. Thus the extreme left and right hand terms in equation (3) balance one another. The remainder of the equation then takes the form:

$$v_g\left(\frac{\pi}{4}D^2\right)\frac{\rho_g v_g^2}{2} \sim \alpha\,\frac{\rho_f u_f(\pi/4)d^2}{\rho_f(\pi/6)\delta^3}\,\pi\delta^2\sigma \quad (4)$$

which reduces to

$$\phi \sim \alpha^{1/3}\left(\frac{d}{\delta}\right)^{1/3}\frac{r^{1/3}}{(\beta We_f)^{1/3}} \quad (5)$$

where the quantity $\beta \equiv D/d = (A_{\text{heater}}/A_{\text{jet}})^{1/2}$ is one which Katto *et al.* did not include explicitly in their correlation.

The actual appearance of burnout under the postulated mechanism is as follows: As burnout is approached the lower (horizontal) liquid flow remains intact, and the spray above it is quite fine. Then the lower flow suddenly stops, the spray becomes more coarse, and the angle between the plate and the spray increases. The liquid ceases to absorb kinetic energy and to protect the plate, which is now insulated around its periphery. This model is verified both by Monde and Katto's description and by our own observation of the process in a barbotage analogue which we built in our laboratory.

The work of Katto and Ishii casts some light on the nature of α in our equation (5). They observed that with jets of Freon 113 and trichloroethane there was far less spray splashed out than there was with water. Since water has a value of r almost an order of magnitude greater than the other two fluids, we shall presume that $\alpha = \alpha(r)$. Equation (5) then takes the form:

$$\phi \sim \left(\frac{d}{\delta}\right)^{1/3}\frac{f_1(r)}{(\beta We_f)^{1/3}}. \quad (6)$$

THE EVALUATION OF $(d/\delta)^{1/3}$

The term $(d/\delta)^{1/3}$ in equation (6) presents a serious difficulty; however we can obtain some guidance in its evaluation both from dimensional analysis and from the literature. The functional dependence of δ should be:

$$\delta = \delta(\rho_f, \rho_g, v_g, \sigma, \text{liquid film thickness})$$

where the liquid velocity is probably unimportant and where the film thickness (see Fig. 1) depends on the local radius of the disc. Since most of the disc area lies close to a radius of $D/2$, we can replace the thickness with $d(1-\alpha)/\beta$. There are six variables in length, force, and time, so we look for three dimensionless groups:

$$\frac{\delta\beta}{d(1-\alpha)} = f_2\left(\frac{\rho_f v_g^2 d(1-\alpha)}{\sigma\beta}, \frac{\rho_f}{\rho_g}\right).$$

If the groups (with the exception of ρ_f/ρ_g) obey a power-law relation, then this becomes

$$\left(\frac{d}{\delta}\right)^{1/3} \sim \beta^{1/3}(We_f^a\phi^{2a}\beta^{-a})f_3(r). \quad (7)$$

In 1938, Nukiyama and Tanasawa [5] provided the classical expression for the Sauter mean diameter of droplets formed in spray nozzles. It consisted of two terms, the second of which is a viscous term that is quite negligible in our range of interest. The first term is of the form:

$$\delta \sim \sigma^{1/2}/\rho_f^{1/2}v_g.$$

This expression is missing a length—presumably an appropriate jet diameter, d. With $d^{-1/2}$ multiplied on the left of the expression above, it can be arranged as

$$\left(\frac{d}{\delta}\right)^{1/3} \sim We_f^{1/6}\phi^{1/3}\beta^{-1/6}. \quad (8)$$

This result would give a as 1/6 in equation (7), and no dependence on r. Any handbook or survey article on sprays will offer competing formulas for droplet size in different spray-making configurations, none of which are the same as the one under consideration. They generally yield values of a on the order of magnitude of 1/6 or less and minimal, if any, dependence on r.

Returning to equation (5) with equation (7) we find

$$\phi^{1-2a} \sim f_4(r)/\beta^a We_f^{1/3-a}$$

or

$$\frac{\phi\beta}{f_5(r)} \sim \left(\frac{\beta^3}{We_f}\right)^{(1-3a)/(3-6a)} \quad (9)$$

The Monde and Katto correlation for the axisymmetric jet arrangement leads to $f_5(r) = r^{0.725}$ while Katto and Ishii's result for the plane jet geometry gives $f_5(r) = r^{0.867}$. In the latter case β would be $(A_{\text{heater}}/A_{\text{jet}})^{1/2}$ instead of D/d. Both cases have $a = 0$ so that the β dependence is absent in the equations. Geometry effects evidently remain, but more importantly, the assumptions of power law dependences throughout (that we have also made, in part) might not adequately reflect the true mechanisms. As we will show below, a slight dependence of ϕ on β is present, and it is only approximately true that a is equal to more over the whole range of We_f.

COMPARISON OF CORRELATION WITH DATA

We have plotted the group $\beta\phi/r^{0.725}$ against We_f/β^3 in Fig. 2 using the saturated boiling data that were correlated in [1]. Some error was doubtless incurred by scaling

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†We are being a little casual about averages here. δ is the diameter of the droplet with *average surface energy*. Later we relate δ to a Sauter mean diameter. Since the various averages differ by a constant factor which need not be evaluated, no harm will be done.

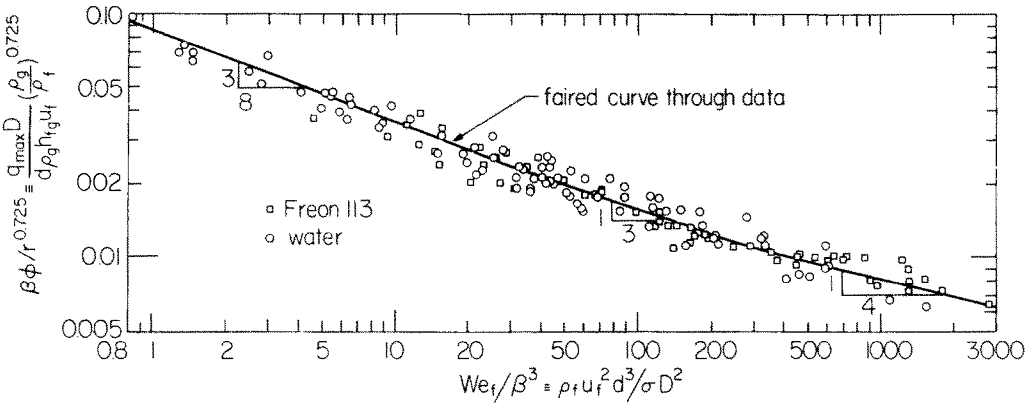

FIG. 2. The data of Monde and Katto under the present scheme of correlation.

numbers from the graph in [1], but this should be small in comparison with the inherent scatter of the points. Figure 2 reveals:

(1) About 94% of the data correlate within $\pm 20\%$ of the best line through their midst. This is a substantially better correlation even than given by equation (1).

(2) The slope of the curve varies from 3/8 on the left through 1/3 in the middle to 1/4 on the right. These slopes give values of a in the droplet diameter relation, equation (7), equal to $-1/6$, 0 and $+1/6$ (Nukiyama's case), respectively. They are reasonable values in the light of what droplet diameter expressions look like in other cases. And it appears that Monde and Katto's choice of $We_f^{1/3}$ was the best single exponent they could have used.

The data of Katto and Ishii for planar jets are less abundant than those of Monde and Katto and they show a little more inherent scatter. Our correlation of those results gives $a = 0$. This collapses our correlation, equation (9), exactly into equation (2), and we get the same numbers as Katto and Ishii did.*

In the present correlations we have taken a to be a constant, although nothing in our dimensional analysis prevents it from varying with r. We have also accepted Katto *et al.*'s power laws for $f_5(r)$. Unfortunately there are only 2 values of r, in each case, with which to fix these functions. (2 of 3 values in the Katto–Ishii paper are almost the same.) Professor Katto has informed us of ongoing high pressure experiments which will greatly extend the range of r. When data from these tests are available, the r-independence of a and the function $f_5(r)$ can be questioned further.

*We are grateful to Mr. Md. Z. Hasan for correlating the Katto–Ishii data for us.

CONCLUSION

The rate-of-energy stability criterion advanced in [3] appears to be applicable in the present problem. Its use results in a predicted correlation which works slightly better than—although it is similar to—the empirical expression given by Monde and Katto. The present analysis suggests two foci for future experimental work. (1) The spray mass flow fraction largely dictates $f_5(r)$. Thus α should be observed as a function of r and of the configuration. (2) Spray droplet sizes should be measured in these boiling configurations to fix "a" *a priori*.

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